and

$$CK' = \sqrt{b^2 + (z^2 \operatorname{cosec} a - a)^2} + z^2,$$

therefore, after reciprocation with respect to the centre C, the radius of the circle in the crescent (Fig. 1) which corresponds to the circle O' of radius z^2, will be equal to

$$\frac{1}{2}\left(\frac{1}{\sqrt{b^2 + (z^2 \operatorname{cosec} a - a)^2} - z^2} - \frac{1}{\sqrt{b^2 + (z^2 \operatorname{cosec} a - a)^2} + z^2}\right)$$

or

$$\frac{\tan^2 a \cdot z^2}{(z^2 - z_1^2)(z^2 - z_2^2)}, \tag{4}$$

where z_1^2 and z_2^2 are the roots of the equation

$$(z^2)^2 - \frac{2a}{\cos a}(z^2) + (a^2 + b^2)\tan^2 a = 0,$$

considered as a quadratic equation.

In this equation a is given, being one-half the angle which the two given circles make with each other. a and b are also known since they are determined by the condition that the centre C (Fig. 2) of reciprocation was so chosen that the two straight lines OST and $OS'T'$ transform into the two circles OCQ and $OAPB$ (Fig. 1).

Hence the radius of the reciprocal of any nth circle will be

$$\frac{\tan^2 a \cdot h^{\pm 2n} z^2}{(h^{\pm 2n} z^2 - z_1^2)(h^{\pm 2n} z^2 - z_2^2)}.$$

Therefore the sums of the radii and areas of the circles in the crescent will be respectively,

$$S_r(z) = \sum_{+\infty}^{-\infty} \frac{\tan^2 a \cdot h^{2n} z^2}{(h^{2n} z^2 - z_1^2)(h^{2n} z^2 - z_2^2)}, \tag{5}$$

$$S_A(z) = \sum_{+\infty}^{-\infty} \frac{\pi \tan^4 a \cdot h^{4n} z^4}{(h^{2n} z^2 - z_1^2)^2 (h^{2n} z^2 - z_2^2)^2}. \tag{6}$$

II.

In this section we shall calculate $S_r(z)$ in terms of σ-functions.

From §8, (6) of Weierstrass and Schwarz's *Elliptic Function Formulas* (Edition 1885), we have

$$\frac{\sigma'}{\sigma}(u) = \frac{\eta}{\omega}u + \frac{\pi i}{2\omega}\left(\frac{z^2+1}{z^2-1} + \sum \frac{2h^{2n} z^{-2}}{1 - h^{2n} z^{-2}} - \sum \frac{2h^{2n} z^2}{1 - h^{2n} z^2}\right), \tag{7}$$