$$N_u A_{ul}(\nu) = \frac{\gamma_{ul}^T/4\pi^2}{(\nu - \nu_0)^2 + (\gamma_{ul}^T/4\pi)^2} N_u A_{ul}. \tag{4.65}$$

INHOMOGENEOUS BROADENING For inhomogeneous emission processes that lead to a Gaussian distribution of emitting frequencies, we described how specific portions of the population density N_u contribute to different portions of the emission linewidth. These differences were shown to occur, for example, in Doppler broadening for atoms that are moving in different directions with respect to the location from which the emission is observed. Each of those separate directions was described as having different velocities v_x with respect to the observer. For an incrementally small velocity segment Δv_x ranging from v_x to $v_x + \Delta v_x$, all of the atoms within that segment will radiate with their homogeneously broadened emission linewidth. But since it represents only a small portion of the entire frequency distribution if Doppler broadening is dominant, this homogeneous broadening is not apparent when the entire emission line is observed. What is apparent is the consequence of all of the individual velocity segments adding up over the linewidth, yielding the Gaussian shape that is determined by the population distribution according to velocity as shown in Figure 4-12.

We can thus express the population distribution per unit frequency as a function of the frequency ν as follows:

$$N_u(\nu) = C_2 \exp\left\{-\left[\frac{4 \ln 2(\nu - \nu_0)^2}{\Delta\nu_D^2}\right]\right\}, \tag{4.66}$$

where the Gaussian profile was obtained from (4.60). The normalizing constant can be derived by assuming that the sum of all of the population densities $N_u(\nu)\,d\nu$ radiating at specific frequency intervals $d\nu$ over the Doppler-broadened emission linewidth must equal the total population density N_u in level u:

$$N_u = \int_0^\infty N_u(\nu)\,d\nu. \tag{4.67}$$

Carrying out this normalization integral using (4.66) for $N_u(\nu)$, we find that

$$C_2 = 2\sqrt{\frac{\ln 2}{\pi}} \frac{1}{\Delta\nu_D} N_u; \tag{4.68}$$

as a result, the population density per unit frequency is

$$N_u(\nu) = 2\sqrt{\frac{\ln 2}{\pi}} \frac{1}{\Delta\nu_D} N_u \exp\left\{-\left[\frac{4 \ln 2(\nu - \nu_0)^2}{\Delta\nu_D^2}\right]\right\}. \tag{4.69}$$

Thus we have