

class of human beings and ‘ m ’ the class of all mammals, then hIm is a true sentence while mIh is a false sentence. Since every class is regarded as a subclass of itself (though not a proper subclass), also hIh and mIm are true sentences. The sequences of symbols xIy are examples of expressions that contain variables and that become sentences when we substitute designations for specific objects (in the case of this language, names of specific classes) for the variables. They are thus called *sentential functions* and act just like mathematical functions such as $x + y$, which have a specific value only when we substitute for the variables names of specific numbers like ‘2’ for x and ‘5’ for y to yield $2 + 5 = 7$.

There are other ways than substitution for the variables to lead to sentences from sentential functions: we can form the *negation* of a sentential function F by preceding it by the symbol ‘ \neg ’ to form $\neg F$ (read “not F ”), and we can form the *disjunction* of two sentential functions F and G by joining them by the symbol ‘ \vee ’ to form $F \vee G$ (read “ F or G ”). Finally, we can form the result of *universal quantification* of a sentential function F with respect to a variable x that may occur in F , by preceding F by ‘ $\forall x$ ’ to form $(\forall x)F$ (read “for all x , F ”).⁷

The following are examples of sentential functions constructed by these three means:

- (6) $\neg xIx$, the negation of xIx , which expresses for any given class x that x is not included in x ;
- (7) $xIy \vee yIx$, the disjunction of xIy and yIx , which expresses for any given classes x and y that x is included in y or y is included in x ;
- (8) $(\forall x)xIx$, the universal quantification with respect to x of xIx , which expresses that for *all* classes x , x is included in x .

Of these three, only (8) expresses a completed sentence, which in this case is true. In general, we can construct sentences from sentential functions by applying the universal quantifier to the variables that occur in them. Thus, for example:

- (9) $(\forall x)(\forall y)(xIy \vee yIx)$ expresses that, for all classes x and y , either x is included in y or y is included in x .

That sentence is false, since there are classes neither of which is included in the other (e.g., the class of living humans and the class of humans who ever visited France). If we precede the sentence in (9) by the symbol ‘ \neg ’ we thus obtain a true sentence. To form more interesting sentences it is